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A lattice theory approach to the structure of mental models

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Lattice theory is proposed to provide a formalism for the knowledge base used as a mental model by the operator of a complex system. The ordering relation '$\geqslant$' is interpreted as 'is caused by', and the lattice becomes a representation of the operator's causal hypotheses about the system. A given system can be thought of causally in different ways (purposes, mechanics, physical form, etc.). Each gives rise to a separate lattice. These are related to each other and to an objective description of the structure and function of the physical system by homomorphic mappings. Errors arise when nodes on the mental lattices are not connected in the same way as the physical system lattice; when the latter changes so that the mental lattice no longer provides an accurate map, even as a homomorphism; or when inverse one–to–many mapping gives rise to ambiguities. Some suggestions are made about the design of displays and decision aids to reduce error.

1. Introduction

It is often suggested that operators control complex sytems by forming mental models of the system, and it is their mental model that allows them to control a system too large for all variables to be monitored. They use observations of some subset of variables to decide what action must be taken by using their mental model to predict future states of the entire system. (See, for example, many papers in Sheridan & Johannsen (1976), Rasmussen & Rouse (1981), and Moray (1979). The concept of mental models has gained widespread acceptance, but there is no formalism to represent such models. The main purpose of the present paper is to propose a general formalism for mental models.

2. Mental models of systems operation

The ideas here were inspired by a passage in Ashby (1956) where he defines a model and relates it to lattice theory: 'We can now see much more clearly what is meant by a "model"...(Earlier we saw how) three systems were found to be isomorphic and therefore capable of being used as representations of each other...The model will seldom be *iso*morphic with the biological system: usually it will be a homomorphism of it...The model itself is seldom regarded in all its practical detail: usually it is only some *aspect* of the model which is related to the biological system. Thus what usually happens is that the two systems, biological and model, are so related that a homomorphism of one is isomorphic with a homomorphism of the other...The higher the homomorphisms are on their lattices, the better or more realistic will be the model.' (Ashby 1956, p. 109.) Ashby was discussing the way in which biological systems could be modelled by cybernetic physical machines. In this paper I discuss the inverse: how physical machines are modelled in the mind and brain.

Isomorphism and homomorphism are concepts to do with *mappings* between the elements of sets, and this in turn leads naturally to a discussion of lattice theory. Ashby proposes that any large system with interacting subsystems can be represented as a lattice, and that the elements of the lattice represent the decomposition of the system into (relatively independent) subsystems by a series of many–one mappings (Ashby 1956). If a mental model is a lattice mapping of the properties of the real physical system into a knowledge structure in the mind, what kinds of mappings occur, what is the structure of the lattice, and what implications are there for what information can be known and used by the operator?

3. Abstraction hierarchies

Rasmussen (1986) has suggested that complex systems are understood at several levels which together comprise an abstraction hierarchy. At the lowest level there is extremely detailed knowledge about the physical form of a system (nuts, bolts, switches, physical layout). Above that, the system can be thought of as subsystems made up of collections of physical form components, the level of physical function (pumps, valves and heating circuits). Above that is a level of generic function (heating, temperature control, pressure control, parts handling, assembly and maintenance). Above that is the level of abstract function (mass and energy flows, information flows and productivity). Finally, the system can be thought about in terms of its overall goal or purpose.

He suggests that as people perform different tasks they think about the system at different levels. For example, fault diagnosis usually requires concepts at the levels of generic and then physical function, whereas repair and maintenance require understanding at the level of physical function and physical form. The amount of detail varies enormously at the different levels, and what can be thought about depends on the level at which the person is thinking because there is a different decomposition of the system at the different levels. At the highest levels nearly all detail is lost.

This description of a system in terms of an abstraction hierarchy bears a marked resemblance to Ashby's suggestion that a model is a lattice mapping. We can think of a mental model as a series of lattices, each of which contains a certain kind of information, and which map onto one another as homomorphs (and occasionally, in special cases, as isomorphs).

4. A lattice theory of mental models

For a full discussion of lattice theory see, for example, Birkhoff (1948), Szasz (1963) or Skornjakov (1977). A lattice is a partially ordered set (poset) of elements ordered by the relation '$\geqslant$'. For all the sets with which we are concerned, the set is closed, as physical systems have a finite number of parts. The set has a least element (0) and a greatest element (1). Elements of the set can be represented as points on a diagram. Elements higher on the lattice are composed of unions of elements lower on the diagram. If any two points are joined by a rising line they are related by the order relation $\geqslant$. Only elements so connected are related (no horizontal lines are permitted). A new lattice can be formed from an existing lattice by means of mapping rules. If the mapping is '1–to–1 and onto', then the new lattice is said to be isomorphic with the original lattice. If the mapping is many to one, the mapping is homomorphic. We agree with

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Ashby that a lattice formed from a previous lattice by mapping is by definition a *model* of that lattice. In general, inverse mappings can be performed, but will only recover the original lattice for isomorphisms, not homomorphisms. Figure 1 shows examples of graphs that include lattices and examples of mappings.

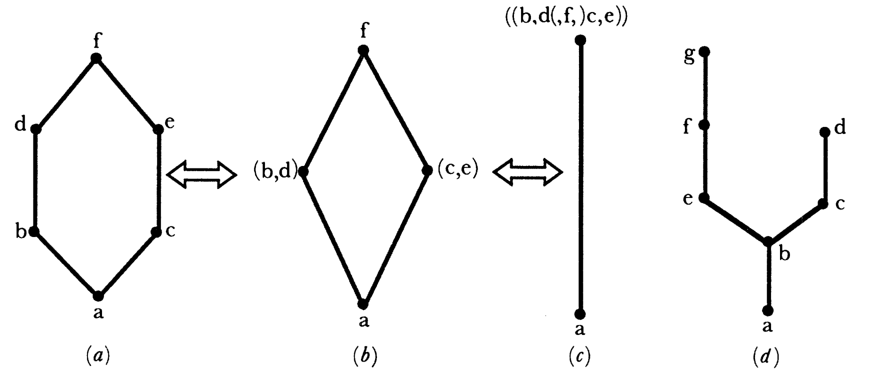

FIGURE 1. Examples of lattices and mappings. Parts (*a*)–(*c*) represent a lattice with progressive simplification by means of homomorphic mapping. The arrows represent mappings, which can be in either direction. Part (*d*) is not a lattice because there is no single greatest element to which the topmost elements are both connected.

The ability of lattice theory to represent mental models of systems knowledge arises when we interpret the order relation $\geqslant$. The algebraic interpretation is 'greater than or equal to', but the notation supports any interpretation that preserves partial ordering. Consider the ways in which a person may think about the functional and causal relations between parts of a system. Classically (in the sense of going back at least to Aristotle), four kinds of cause have been distinguished. A switch may cause a pump to operate because it is the 'on' position (formal cause), because it closes a pair of contacts (material cause), because it allows current to flow through the pump (efficient cause), or because we need cooling (final cause). But causality is an ordering relation in physical systems. A switch causes a pump to operate, which causes coolant to flow, which causes heat removal, which controls the reaction temperature, and so on. We can represent the causal relations between the parts of a system by a lattice, interpreting '$\geqslant$' to mean 'is caused by'. If two elements on the lattice are connected by a line, the lower one is the cause of the upper, and the upper one the effect of the lower.

We start with an objective lattice description of the real physical relations between the parts of the system as described in engineering specifications, or (which ideally we take to be equivalent) as discovered by an exhaustive examination of interactions among the physical components of the real plant. This lattice we will call the physical system lattice (PSL). In so far as an operator's mental model is isomorphic to the PSL, just to that extent is it a complete model of the physical system; and just to that extent will the mental model's predictions exactly match the output of the different parts of the physical system when it is provided with system inputs and parameter values. In general, however, the operator's knowledge will be imperfect for at least two reasons. First, if the system is large, it may simply be impossible for the operator to scan and remember the displayed values of the system variables so as to acquire a perfect knowledge of the system relations. (Indeed some of them may not be displayed.) Second, and perhaps more importantly, the abstraction hierarchy suggests that for many purposes mental models will be homomorphs, not isomorphs of the physical system.

The higher the level of the abstraction hierarchy at which a person thinks about the system, the fewer the elements there are to think about. A 'cooling system' may contain several 'pumps'. A 'pump' may contain several 'glands'. A 'gland' may contain several 'seals'. Thus it is advantageous for an operator to think about a system as high up the hierarchy as possible to reduce his mental workload and the amount of data he must carry in his working memory. The higher levels of the abstraction hierarchy are formed from the lower levels by many-to-one mappings that develop during formal training or informal experience with the plant. That is, higher levels of abstraction are homomorphs of lower levels. They preserve the causal relations between subsystems but with a loss of detail.

Suppose that different kinds of causes may give rise to different lattices. Each cause (formal, material, efficient, or final (that is, purpose)) can provide a complete description of the system in its own terms. These descriptions are complementary, not mutually exclusive. Each can be derived as a lattice (formal cause lattice (FCL), material cause lattice (MCL), efficient cause lattice(ECL) and purposive cause lattice (PCL)) by an appropriate mapping from the PSL; and each has its own abstraction hierarchy. In practice each will be defective in a different way. For example, one may know that a particular circuit is present to provide cooling (final cause) and know what values of the display show that it is working and what controls switch it on or off (formal cause), but not know what mechanism is involved or its underlying physical principles (material and efficient cause). In such a case the FCL and PCL lattices will contain elements not present in the posets of the MCL and ECLS. Other examples will occur to the reader.

5. Mental models as sets of lattices

We see therefore that several kinds of knowledge (the causal lattices) can be obtained from knowledge about the original physical system. These causal lattices are derived from the PSL by homomorphic mappings. To complete the picture we assume that they can be mapped into one another. That is, they are homomorphs of one another. This allows operators to change the way in which they think about the problem as well as the level at which they think about the system. Some of the relations among the elements of the mental model are shown in figure 2.

We cannot here consider how learning mechanisms lead to the construction of the lattices and the original mappings. Those features of the theory have yet to be fully worked out, although there seem to be no difficulties in using what is known of induction, concept formation, probability learning, etc. to account for them. But if we examine a steady state of the mental model we can see both the virtues of such a model and also how such a model might lead to error and the inability to manage a faulty system.

Suppose the system operator observes a signal which shows that some component is in an unusual state. This is equivalent to entering one of the lattices at a particular node. Assume that the operator is currently thinking about the component in terms of its purpose. The response to noticing the unusual state will be to ask which other component has as its purpose the control of this component? That question can be answered by going down the PCL lattice from the component in question to the next lower elements on the lattice. If one of them is itself giving an unusual reading, the process is repeated. At some level there will be no unusual component, and the operator will conclude that the lowest greater element above this which has been found abnormal is the 'cause' of the problem. To manage the fault, he must now move to the MCL or ECL lattice, to understand the nature of the physical abnormality that underlies the

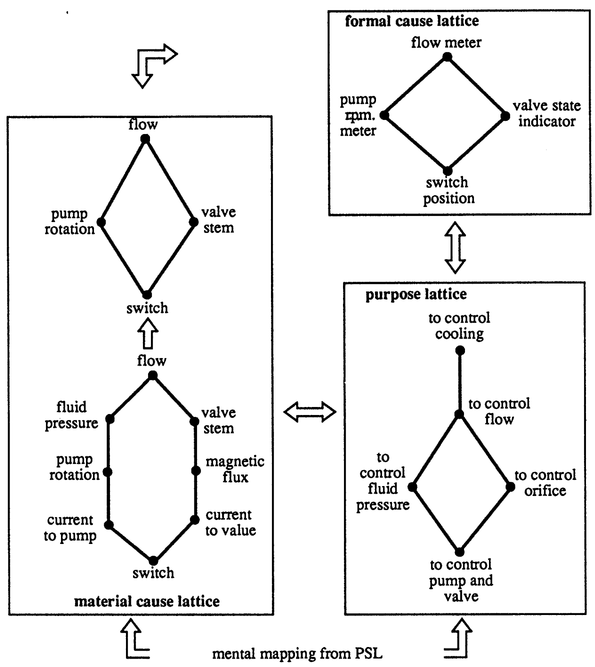

Figure 2. Examples of causal lattices and their simplification and relation by means of mappings. The physical system lattice (PSL) is not shown, but represents part of a cooling system operated by a double pole switch that operates both a pump and an electromagnetic valve to supply cooling water. Note that the existence of mappings does not guarantee that an equivalent node will be available when moving from one lattice to another.

abnormal reading. A mapping carries him into the appropriate lattice at the level where the abnormal PCL was identified, and on this new lattice he once again travels up and down the lattice seeking the lowest level at which there is an abnormal element. That element is the appropriate explanation for the fault. By travelling along the paths of the lattice, and by moving from lattice to lattice, a number of strategies may become apparent for managing the fault. It may be that on returning to the original lattice at its original entry point it is possible to find another component below the one which was abnormal, which is inactive, but which has the same purpose. By then mapping back onto the FCL, the way to switch on that component will be found. The result will be that the operator turns on a redundant standby component and the fault is brought under control.

The multiple representation of knowledge allows the operator to think in a new way about a problem. If he travels all the paths of a particular lattice and does not find any relevant information for the task in hand, he will be forced to try a different lattice. For example, if as a result of an alarm he checks all controls (FCL) and cannot find any abnormal settings, he will have to think about other reasons for the alarm being on, which will suggest a move to one of the other causal lattices. On the other hand, suppose that following the alarm he asks himself a series of questions about the component such as, 'What is it for? What turns it on? Where is its value displayed? What is the underlying physics?' he is staying at the same level of the abstraction hierarchy (same height on the lattice) but moving from lattice to lattice. If no satisfactory information is forthcoming, he will have to change the level of abstraction and try again. In this case exhausting a particular strategy forces him to change levels in each lattice rather than changing from lattice to lattice.

Given a set of switching rules to send the operator from lattice to lattice, such a mental model provides an economical and efficient representation of different ways of thinking about the real system: the PSL has been mapped into a variety of representations each of which provides many levels of abstraction (which provides mental economy and reduces workload if the appropriate level is chosen) and a rich variety of systematically related complementary ways of thinking about a problem (which allows detail when needed). How then does it account for error, what suggestions does it make for aiding risk management?

6. Mental models as sources of error

In almost all discussions of the role of humans as operators of complex systems statements are made that training, displays, etc., must support the acquisition by the operator of a correct mental model of the system. The implication is that should this not happen, errors will occur and efficient fault management will not be possible. How are these claims reflected in the properties of lattice theory?

First, recall that the structure of a lattice is strongly constrained. No horizontal links are possible. Hence variables at the same level in the lattice cannot be 'compared'. Only variables that are related by '$\geqslant$' can be understood as causes or effects of one another. If in the original learning mistakes are made about the structure of the lattice, these mistakes may make it impossible to understand certain components as linked in cause–effect relations. Those relations are literally unthinkable, as only lattice comparable elements have a conceivable relationship.

Secondly, one lattice is usually derived from another by homomorphic rather than isomorphic mapping. This has the advantage of reducing workload for normal situations, but means that the inverse mapping is one–to–many, which means in turn that there is bound to be uncertainty in choosing a route downwards on the lattice. Upward homomorphism reduces 'variety' in Ashby's phrase, and purchases economy of effort at the price of loss of Requisite Variety (Ashby 1956). Hence, there is a possibility that the pursuit of efficiency in the operation of normal systems will be effected at the cost of ambiguity when handling abnormal situations. Note that whenever we construct a physical system that is so complex that it forces the operator to make use of lattice homomorphism to handle it we guarantee that this problem will exist.

Thirdly, there is only a homomorphism, not an isomorphism, between the four kinds of causal lattice. It may be that a person has no structure in the lattice that represents formal relations (FCL) and corresponds to the way in which he or she thinks about the purposes of subsystems (PCL). Things that can be thought about one way cannot then be thought about in another easily. But the latter may be the only way, for a given PSL, to solve the problem. The constraints imposed by lattice structure and homomorphic mapping has the effect of literally making certain things unthinkable, and the more solidly the model becomes entrenched, the less is it possible to think those thoughts. Only relations that are represented on at least one lattice, can be thought about at all, and only those represented on several can be thought about in different ways. There are rigorous ways of examining the PSL and predicting which relations are the natural ones to be embodied in the mental lattices, but they cannot be considered here (for example, see Conant (1976)).

Finally, because of the ordering requirements of a lattice, closed loop structures cause a particular difficulty, because they contain components that are both above and below each other on the lattice, and that is prohibited. I have suggested elsewhere how this might be represented in a lattice framework, and the expected psychological consequences for people trying to diagnose and manage faults containing closed loops (Moray 1988). Briefly, lattice theory predicts that they will either treat the entire closed loop system as an undecomposable black box, or they will try to think about it piecewise as a series of open loop components. There is some empirical evidence for the latter.

What do these results tell us about design for decision aids? Displays and training should indeed help an operator to construct an accurate mental model of the systems with which he or she must work. But they must also be designed so as to make accessible those relations in the system that are likely to be omitted from the causal lattices. This requires an analysis of the PSL to determine its natural decomposability into subsystems (Conant 1976). It is those that are most likely to be preserved in the mental model. In faulty systems couplings change, and displays and decision aids should provide ways to display alternative couplings that are least likely to be built into the mental model, and enable to support the operator when totally new structures emerge. So despite the value of the stable mental model, it must be modified if the PSL changes, although this is a slow process involving learning, whereas movements from lattice to lattice or along paths in a lattice is rapid and voluntary. The systematic analysis of PSL details is a strong strategy for predicting what kinds of faults are most likely to cause severe difficulties for operators when they must perform fault management.

7. Conclusions

There is at present no formalism for describing the structure and content of mental models, although it is widely held that they play a central role in the interaction between humans and complex systems that they operate. The examples given in this paper suggest that lattice theory may be a suitable formalism, and that the properties of posets and homomorphic mappings may provide a representation of how different kinds of causal knowledge are inter-related in mental models, how certain kinds of errors arise, and steps that may help operators in times of emergencies. As formalisms provide strong predictions for empirical tests, this approach may help to move us from purely qualitative accounts of mental models and error to a more powerful basis for empirical studies, as well as providing a basis for the rational design of aids to reduce human error and its impact.

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